Answers to Coursebook questions – Chapter G4

1 **a** There are two distinct parts to X-ray production: the continuous and the discrete parts of the spectrum. The continuous part is produced when electrons that have been accelerated to high speeds strike the metal target atoms and suffer enormous decelerations. Electric charges that are rapidly decelerated emit electromagnetic radiation. The wavelengths emitted vary because the accelerations involved vary. There is a minimum wavelength in the spectrum, obtained when a decelerated electron gives all its energy to a single photon. The discrete part is formed when the accelerated electrons strike electrons inside the metal atoms and eject these from the atoms. Other electrons in the atoms then make transitions to fill the now vacant energy levels, emitting photons in the process. The wavelengths are determined by the energy differences in the levels involved in the transition and are thus characteristic of the metal atoms.

b The maximum kinetic energy of an accelerated electron is 67.0 keV. If the electron gives all this energy to a single photon the photon, then

$$\frac{hc}{\lambda} = 67 \text{ keV} \text{ and so } \lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{67\times10^{3}\times1.6\times10^{-19}} = 1.855\times10^{-11} \approx 1.86\times10^{-11} \text{ m}.$$

2 **a** When the accelerating voltage is increased, the maximum kinetic energy of an electron increases and therefore the energy of the photon emitted increases as well. Thus the frequency increases.

b The kinetic energy of an electron is 30 keV, and so

$$\frac{1}{2}mv^2 = 30\times10^{3}\times1.6\times10^{-19} \Rightarrow v = \sqrt{\frac{2\times30\times10^{3}\times1.6\times10^{-19}}{9.1\times10^{-31}}} = 1.027\times10^{8} \approx 1.0\times10^{8} \text{ m s}^{-1}$$

c $$hf = 30\times10^{3}\times1.6\times10^{-19} \Rightarrow f = \frac{30\times10^{3}\times1.6\times10^{-19}}{6.63\times10^{-34}} = 7.2\times10^{18} \text{ Hz}.$$

3 In the photoelectric effect, photons incident on a metal cause the emission of electrons. In X-ray production the opposite happens: electrons strike a metal and photons (in the X-ray region) are emitted. In the photoelectric effect this happens if the incident photons have a frequency above the threshold, i.e. a wavelength below a threshold value. In X-rays, the photons emitted have wavelengths larger than a certain minimum value.

4 **a** $$\lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{40.0\times10^{3}\times1.6\times10^{-19}} = 3.11\times10^{-11} \text{ m}.$$

b The power produced is $P = VI = 40\times10^{3}\times20\times10^{-3} = 800$ W. The power that is used for heating the anode is then $0.995\times800 = 796$ W, and this is what has to be removed.

5 The two conditions are that

i $2d\sin\theta = n\lambda$ and

ii $\theta_1 = \theta_2$

where θ_1, θ_2 are the angles of the incident and reflected/scattered beams with the crystal plane. The first condition is the condition for constructive interference between beams scattered from atoms in adjacent planes, and the second condition is for constructive interference between beams scattered from adjacent atoms on the same crystal plane.

6 The typical wavelength of X-rays is less than 10^{-10} m, and this is too small for an opening of the order of 10^{-6} m; the X-rays will not diffract.

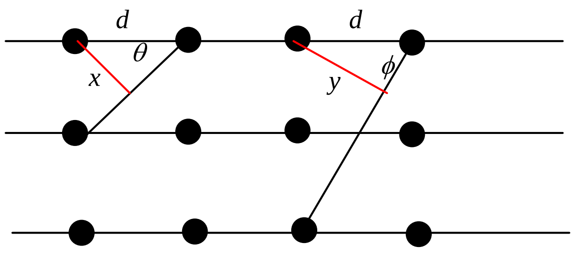
7 $2d\sin\theta = n\lambda$ and so $2\times 4.4\times 10^{-10}\sin\theta = 1\times 6.3\times 10^{-11}$, giving $\theta = 4.1^\circ$ and $2\times 4.4\times 10^{-10}\sin\theta = 2\times 6.3\times 10^{-11}$, giving $\theta = 8.2^\circ$.

8 $2\times d\sin 6.41^\circ = 1\times 6.26\times 10^{-11}$, giving $d = 2.80\times 10^{-10}$ m.

9 **a** For the second order maximum, $2\times 2.5\times 10^{-10}\times\sin\theta = 2\times\lambda$, giving $\theta = 25.4^\circ$.

b We are given that $2\times 2.5\times 10^{-10}\times\sin 12.4^\circ = 1\times\lambda$, and so right away $\lambda = 1.07\times 10^{-10}$ m.

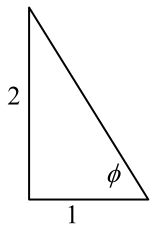
10 We need to know the angles indicated below in order to find the distances *x* and *y*.



Clearly, $\tan\theta = \frac{d}{d} = 1 \Rightarrow \theta = \tan^{-1}1 = \frac{\pi}{4}$, so that $\sin\theta = \frac{\sqrt{2}}{2}$.

Now, $\tan\phi = \frac{2d}{d} = 2 \Rightarrow \phi = \tan^{-1}2$, so that $\sin\phi = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

This follows by considering the triangle below and seeing by Pythagoras that the hypotenuse is $\sqrt{5}$.



a Let the distances between the indicated planes be *x* and *y*.

i

$$x = d\sin\theta = \frac{d\sqrt{2}}{2}$$

and

ii

$$y = d\sin\phi = \frac{2d\sqrt{5}}{5}$$

just by using the definition of the sine in the triangles in the top diagram.

b This shows the enormous complications of actual X-ray diffraction. Many angles at which strong scattering takes place will be found as X-rays are directed at a crystal that is slowly rotated. The many angles will correspond to diffraction from very many different sets of planes, and the analysis of which planes are responsible for which angles is very difficult. A typical result for intensity of scattered X-rays versus angle is shown below, where different colours indicate scatterings from different planes. An actual intensity–angle plot would be much more complicated than this graph.